TIME SHIFT PHENOMENA IN EPIDEMIC MODELS

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MOTIVATION

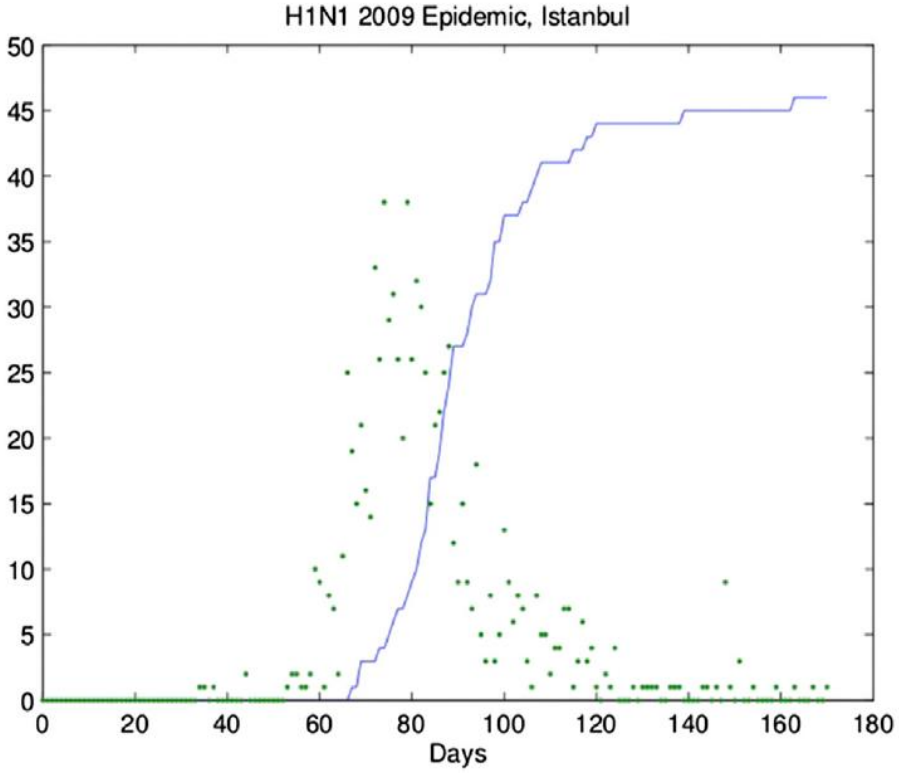
A survey based on the data of the 2009 H1N1 epidemic in Istanbul, Turkey displayed a time shift between the hospital referrals and fatalities [A].

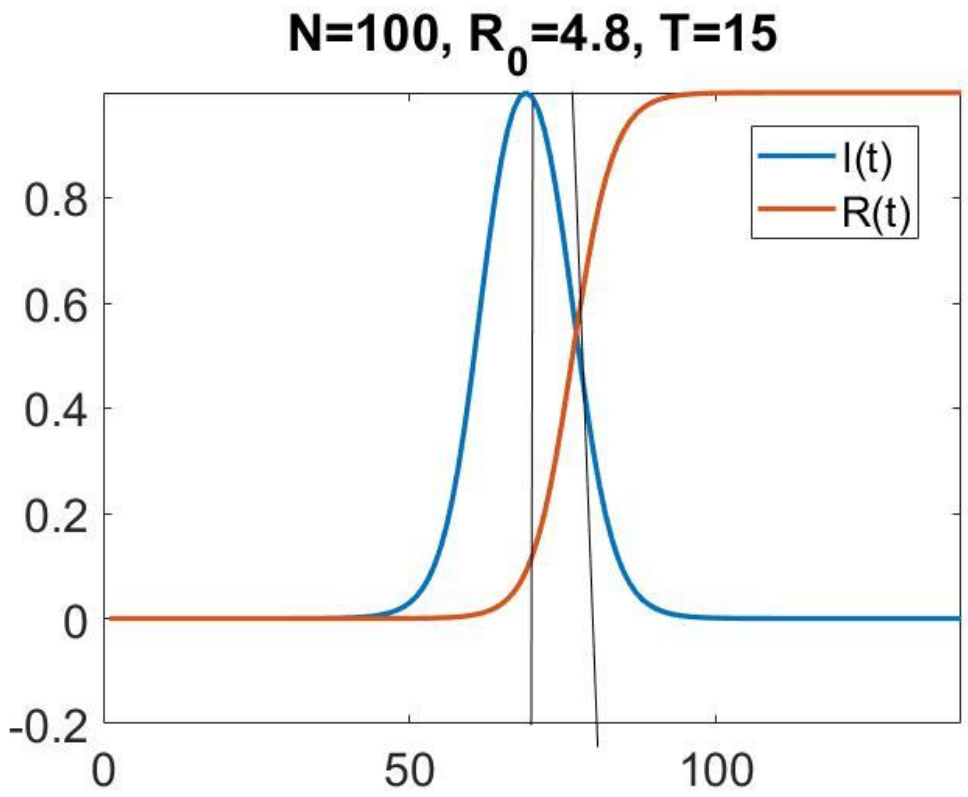
METHOD

Usage of multistage compartmental epidemic models.

[A] Bilge AH, Samanlioglu F, Ergonul O. On the uniqueness of epidemic models fitting a normalized curve of removed individuals. J Math Biol (2015) 71:767–94. doi:10.1007/s00285-014-0838-z

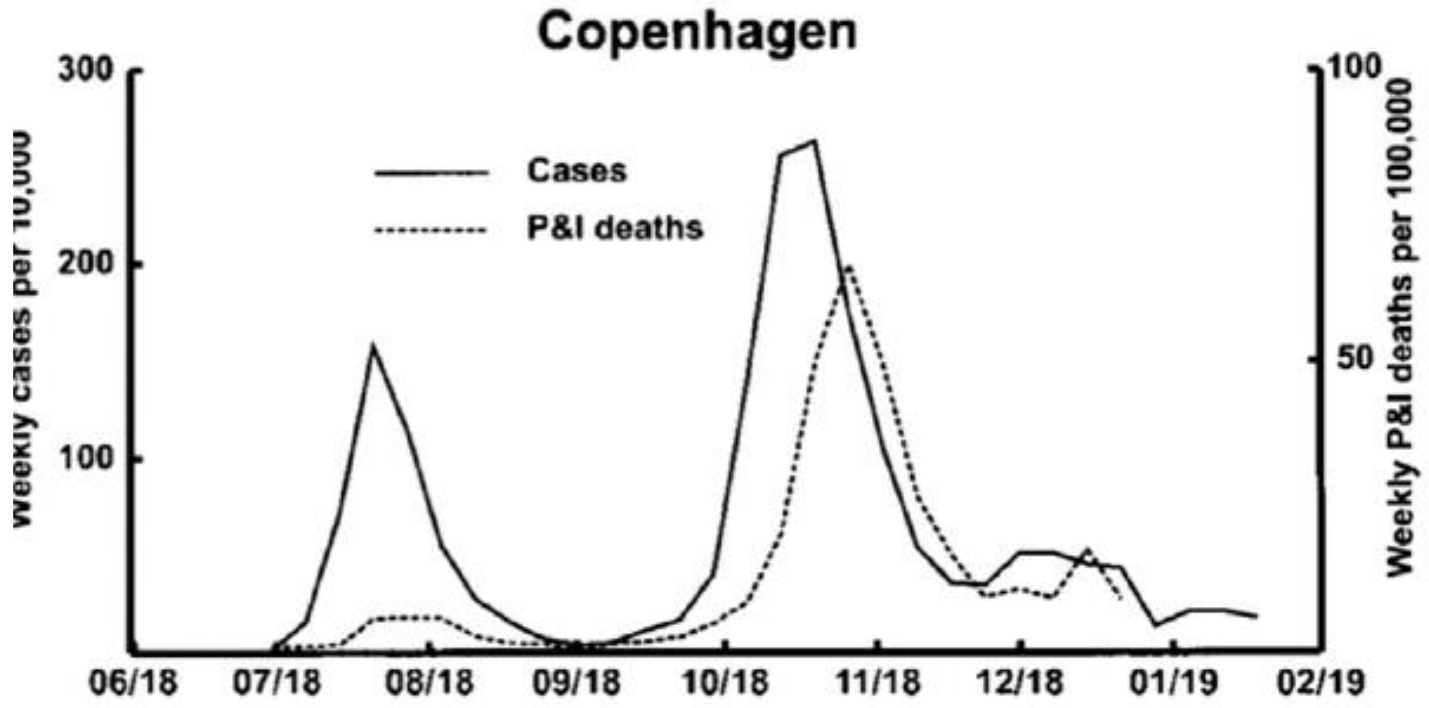
ISTANBUL- A(H1N1) 2009





Daily number of referrals to hospitals and cumulative number of fatalities for the 2009 H1N1 epidemic in Istanbul, Turkey.

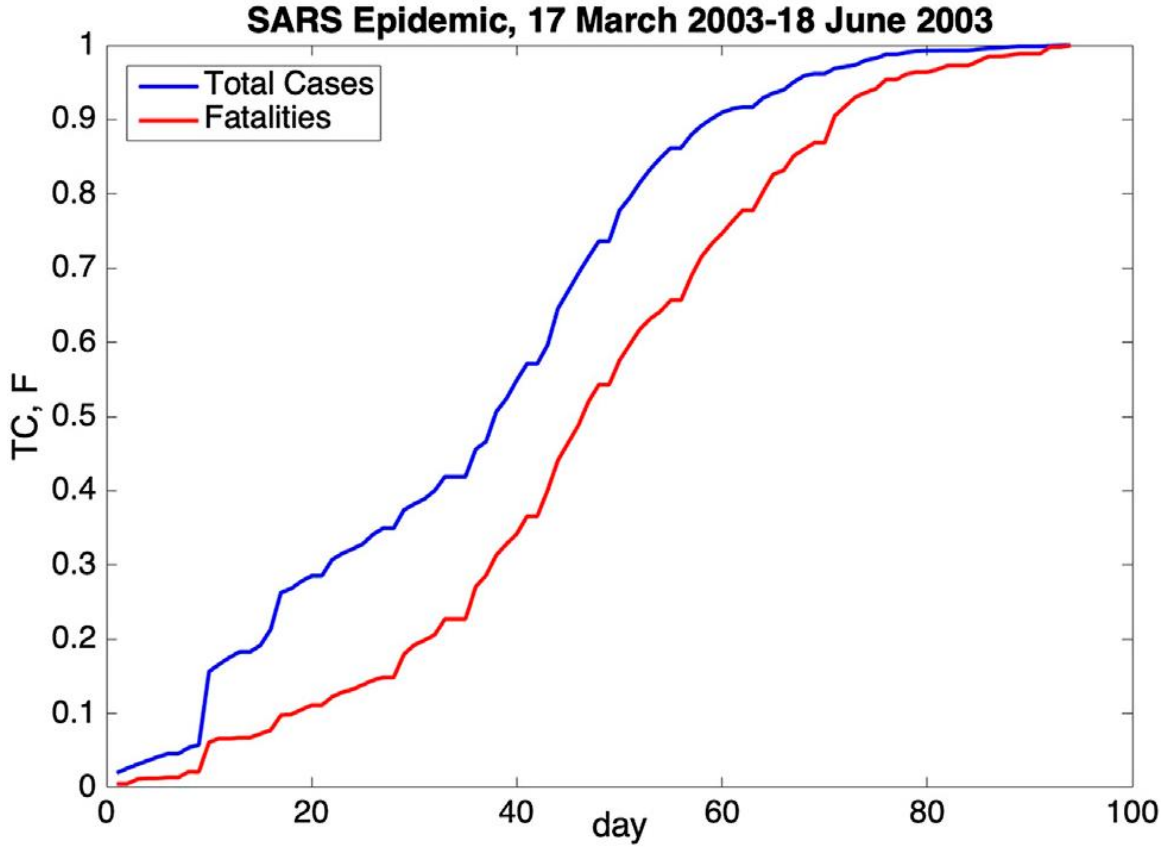
SPANISH FLU 1918-1920



Weekly number of influenza cases and respiratory deaths (pneumonia and influenza) in Copenhagen, Denmark, during 1918–1919 are presented. A shift occurred between the maxima of curves.

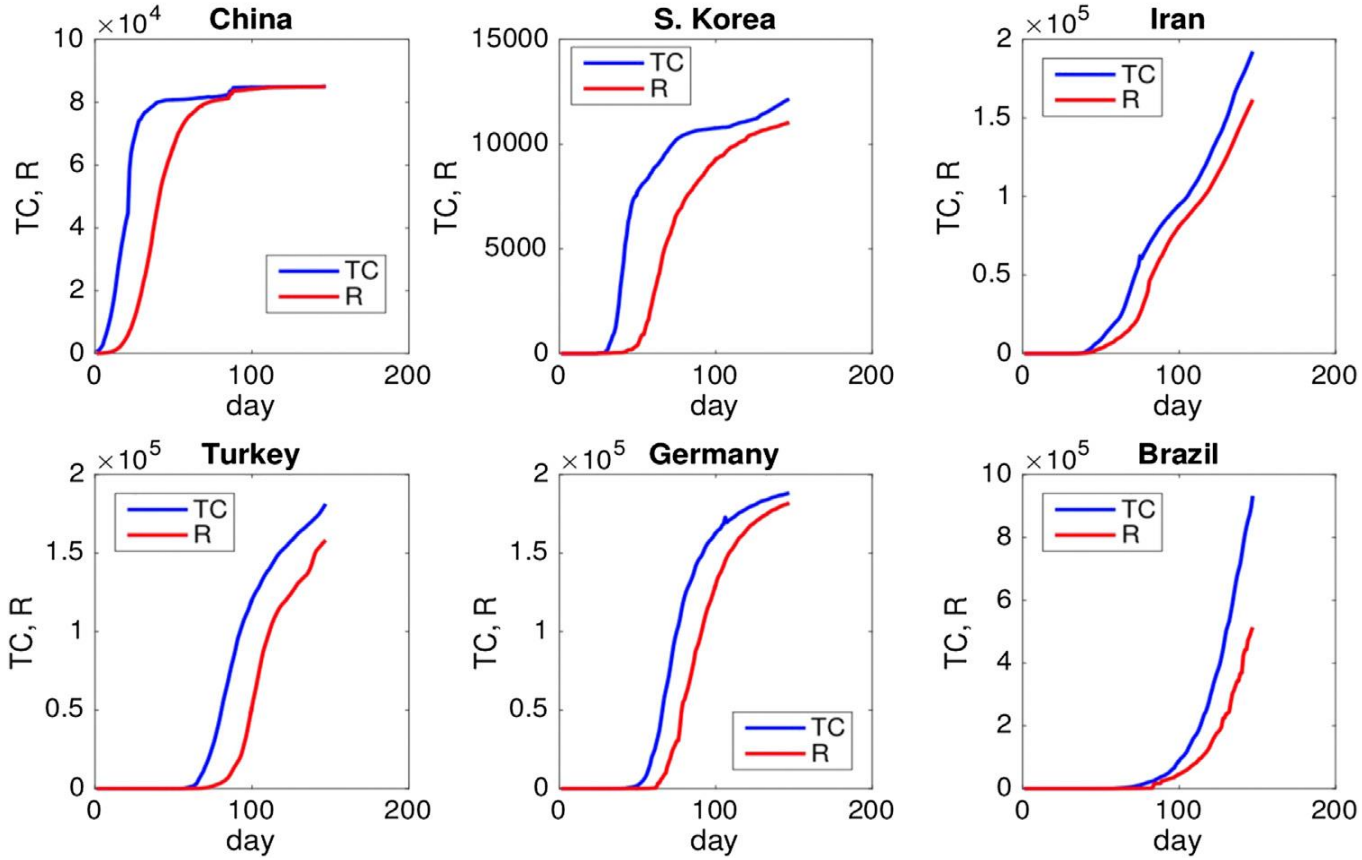
- Andreasen V, Viboud C, Simonsen L. Epidemiologic characterization of the 1918 influenza pandemic summer wave in copenhagen: implications for pandemic control strategies. J Infect Dis (2008) 197:270–8. doi:10.1086/ 524065

SARS 2003



Daily number of total cases and cumulative number of the fatalities for the 2003 SARS epidemic in the world are shown. The data provided by WHO were used in the generation of this figure (https://www.who.int/csr/sars/country/en/) (last access: September 15, 2020). An explicit shift was also observed for the SARS epidemic.

COVID- 19 2020



Graphs of total cases (TC) and removed individuals (R) for selected countries. The dataset of each country is collected according to published official reports and available at the website http://www.worldometers.info/coronavirus/ (last access: June 28, 2020). Updated data are also available at the website http:// epikhas.khas.edu.tr/. The last data in this work were collected on the 28th of June 2020. Data cover the period January 22–June 28, 2020, and in the following, “Day 1”corresponds to January 22, 2020.

SIR-SEIR MODELS

$$SIR: \quad S' = -\beta SI,$$
$$I' = \beta SI - \gamma I,$$
$$R' = \gamma I,$$

$$SEIR: \quad S' = -\beta SI,$$
$$E' = \beta SI - \epsilon E,$$
$$I' = \epsilon E - \gamma I,$$
$$R' = \gamma I,$$

β refers to the disease transmission rate

$1/\gamma$ represents the duration of infection period.

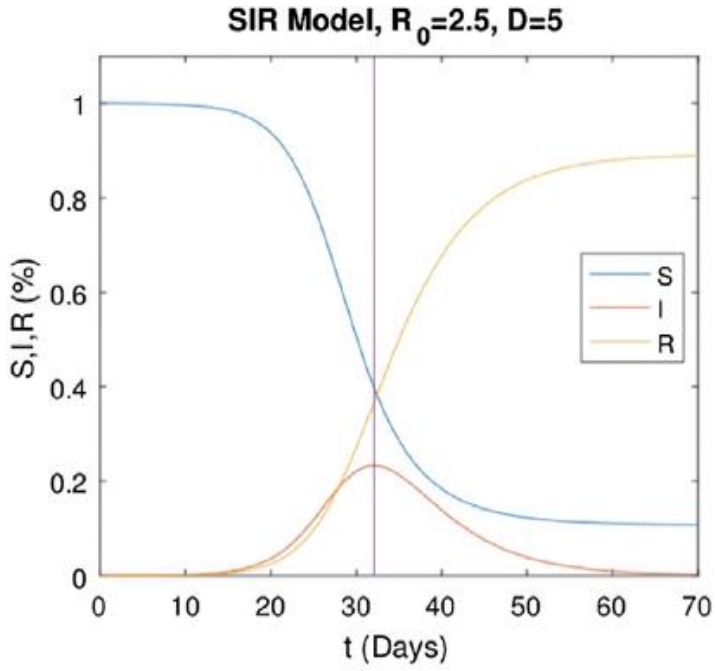
$1/\epsilon$ represents the mean exposed period.

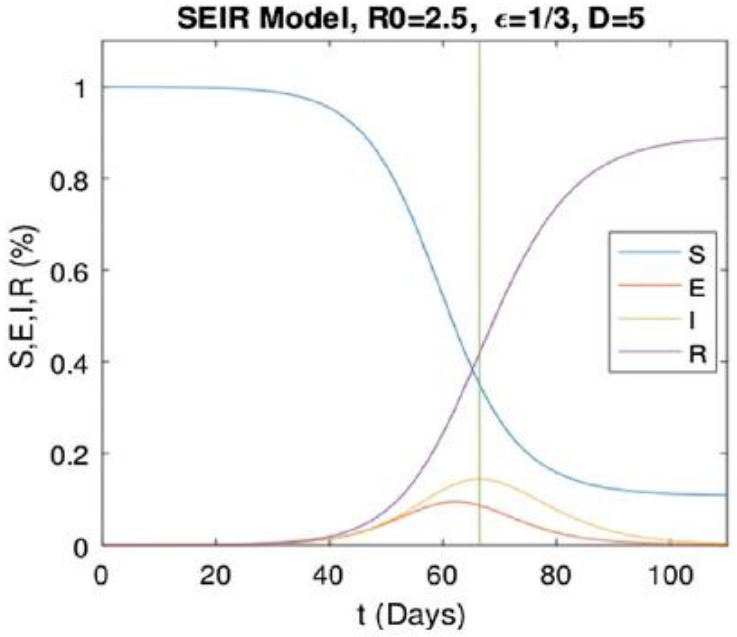
$$S + I + R = 1$$

$$S + E + I + R = 1$$

!! These two models are suitable for mathematical modeling of seasonal diseases, but they fail to reproduce the time shift.

SIMULATION EXAMPLES FOR SIR-SEIR MODELS





Time shift does not occur in classical models.

MULTISTAGE MODELS

- The multistage models (with a realistic number of stages) can be used to represent diseases with different epidemiological stages. But it can also be considered as a mathematical tool for the integral SIR model with Gamma distributed infectious period distribution [L].
- We use this model to explain the time shift.

[L] Lloyd AL. Realistic distributions of infectious periods in epidemic models: changing patterns of persistence and dynamics. Theor Popul Biol (2001) 60: 59–71. doi:10.1006/tpbi.2001.1525.

MULTISTAGE MODELS

$$SJR : J = I_0 + \frac{\beta_1}{\beta_0} I_1 + \cdots + \frac{\beta_N}{\beta_0} I_N,$$

$$S' = -\beta_0 SJ,$$

$$I'_0 = \beta_0 SJ - \gamma_0 I_0,$$

$$I'_i = \gamma_{i-1} I_{i-1} - \gamma_i I_i, \quad \text{for } i = 1, \ldots N,$$

$$R' = \gamma_N I_N,$$

$$SEJR : J = I_1 + \frac{\beta_2}{\beta_1} I_2 \cdots + \frac{\beta_N}{\beta_1} I_N,$$

$$S' = -\beta_1 SJ,$$

$$E' = \beta_1 SJ - \epsilon E,$$

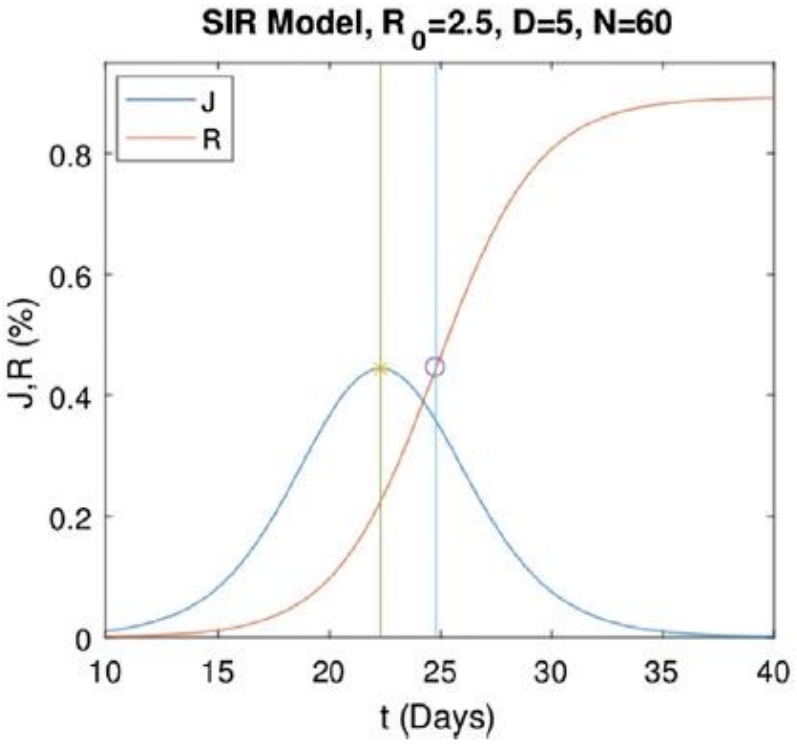
$$I'_1 = \epsilon E - \gamma_1 I_1,$$

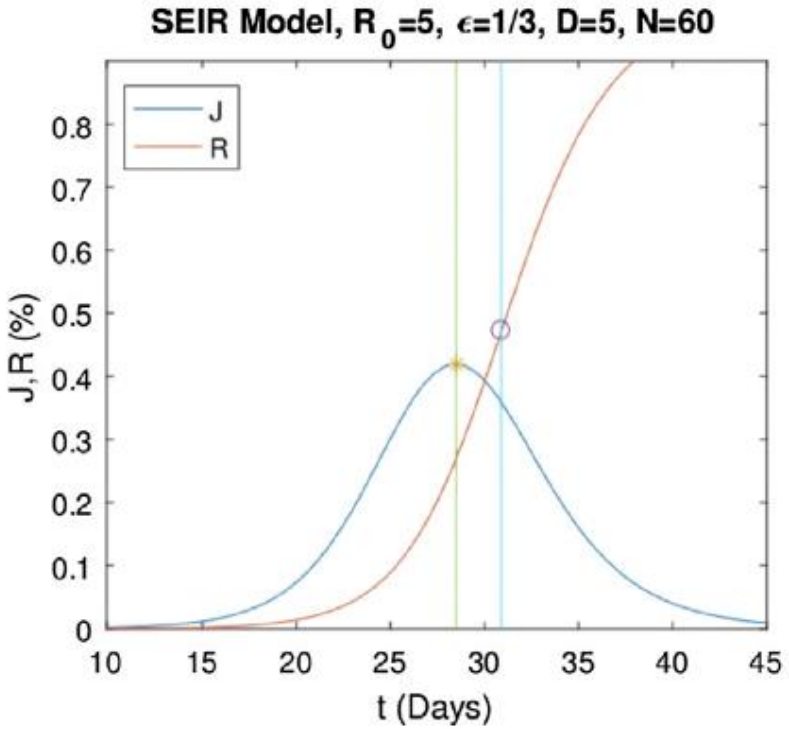
$$I'_i = \gamma_{i-1} I_{i-1} - \gamma_i I_i, \quad \text{for } i = 2, \ldots N,$$

$$R' = \gamma_N I_N.$$

The multistage SIR and SEIR systems with $\beta_0 = \ldots = \beta_n$ and $\gamma_0 = \ldots = \gamma_n$ correspond to the choice of gamma-distributed "Infection Period Distribution" (IPD) in the integral equation formulation of the SIR model [L].

SIMULATION EXAMPLES FOR MULTISTAGE MODELS

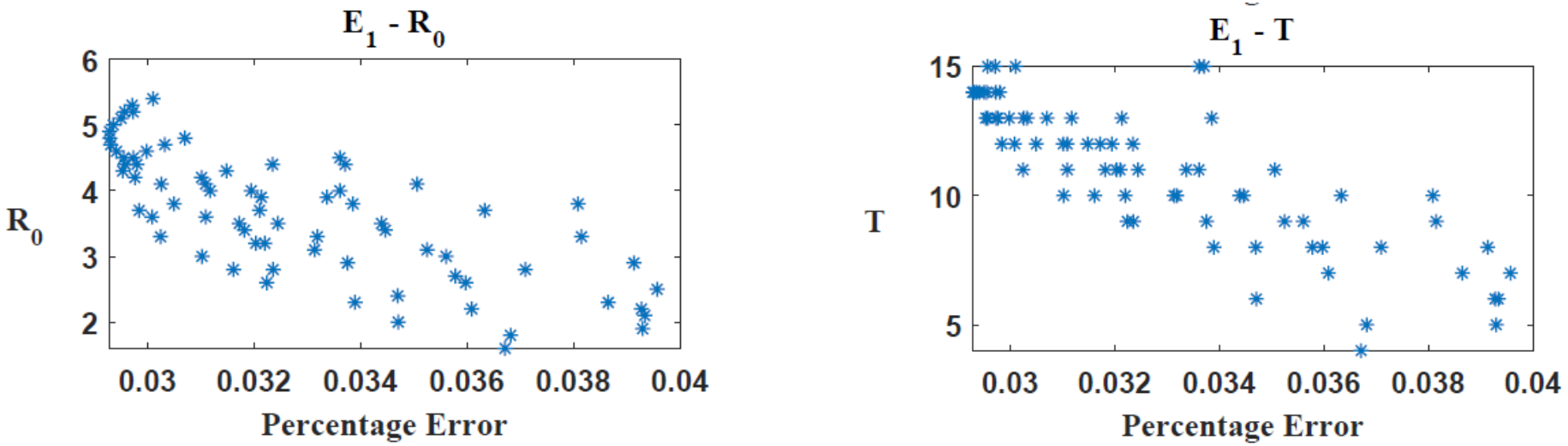




Time shift occur in multistage models.

ESTIMATION OF SHIFT

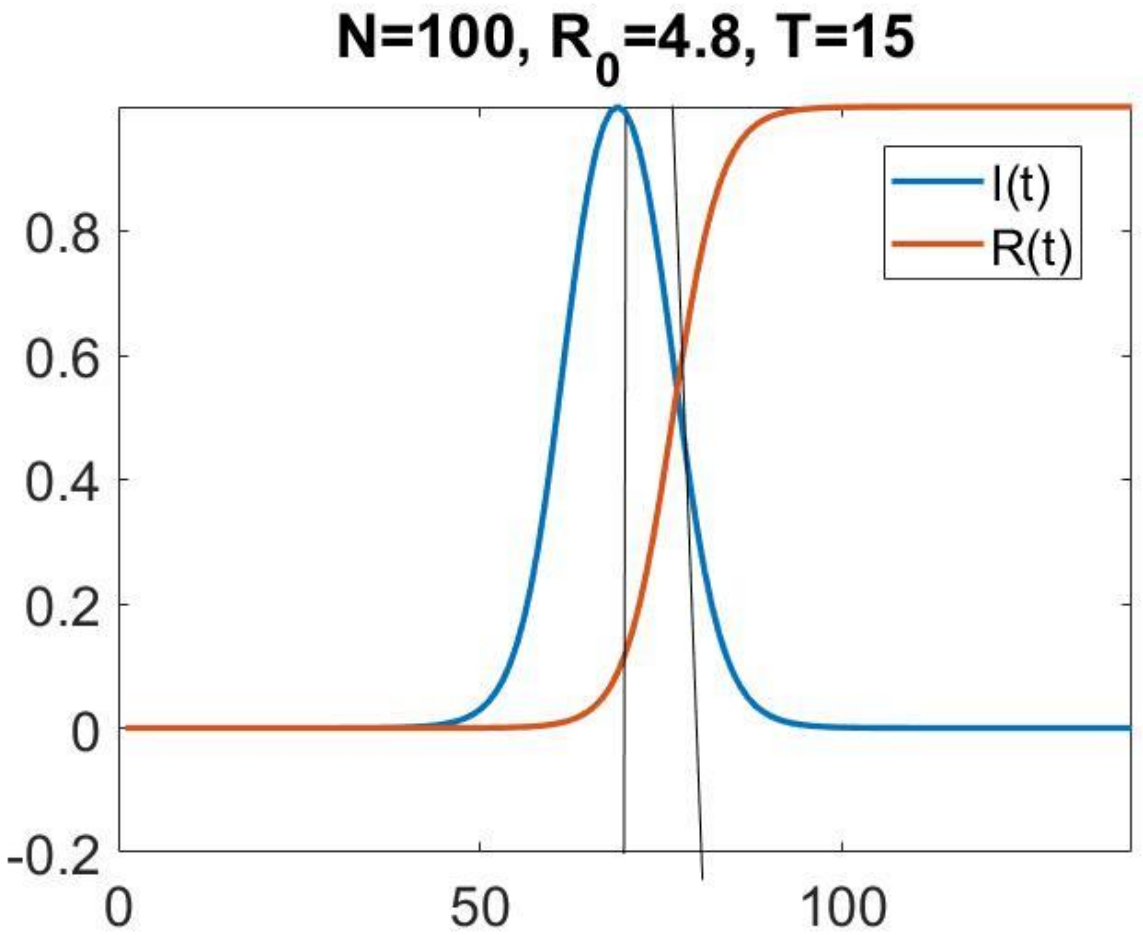
PROPOSITION: For the multi-stage SIR and SEIR epidemic models where $\gamma_i = \gamma$ for all i, the delay between the peak of the total number of infectious individuals and the inflection point of the removed individuals is half of the infection period within a quadratic approximation [U].



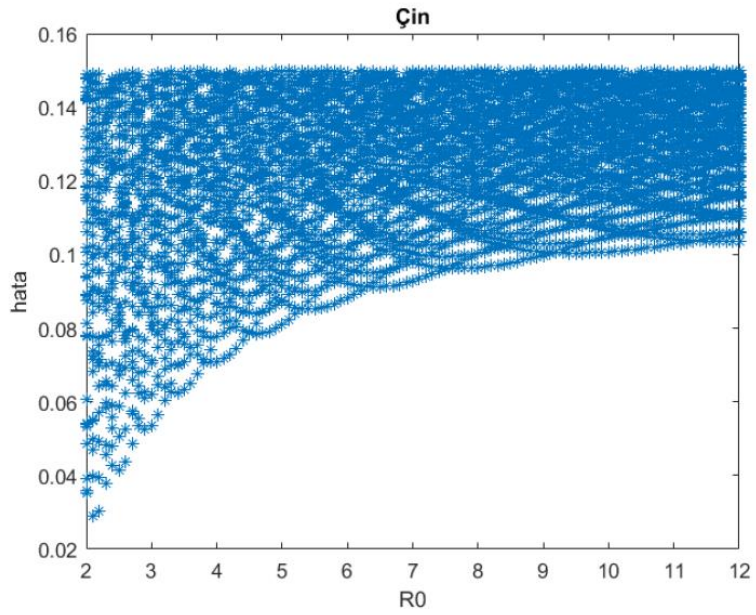
Scatter plots of basic reproduction number and duration of infectious versus L2-error for Istanbul 2009 A(H1N1) data.

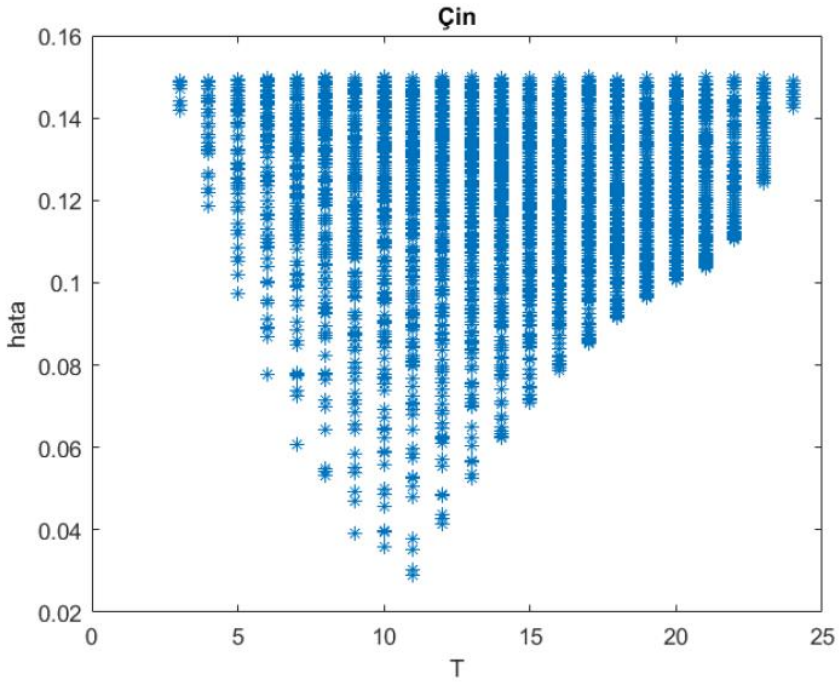
[U] Demirci, A., Dobie, A. P., Bilge, A. H., & Ahmetolan, S. (2020).Unexpected parameter ranges of the 2009 A (H1N1) epidemic for Istanbul and the Netherlands. *arXiv preprint arXiv:2001.10351*.

A SIMULATION FOR ISTANBUL DATA-MULTISTAGE SIR MODEL



COVID-19- MULTISTAGE SIR

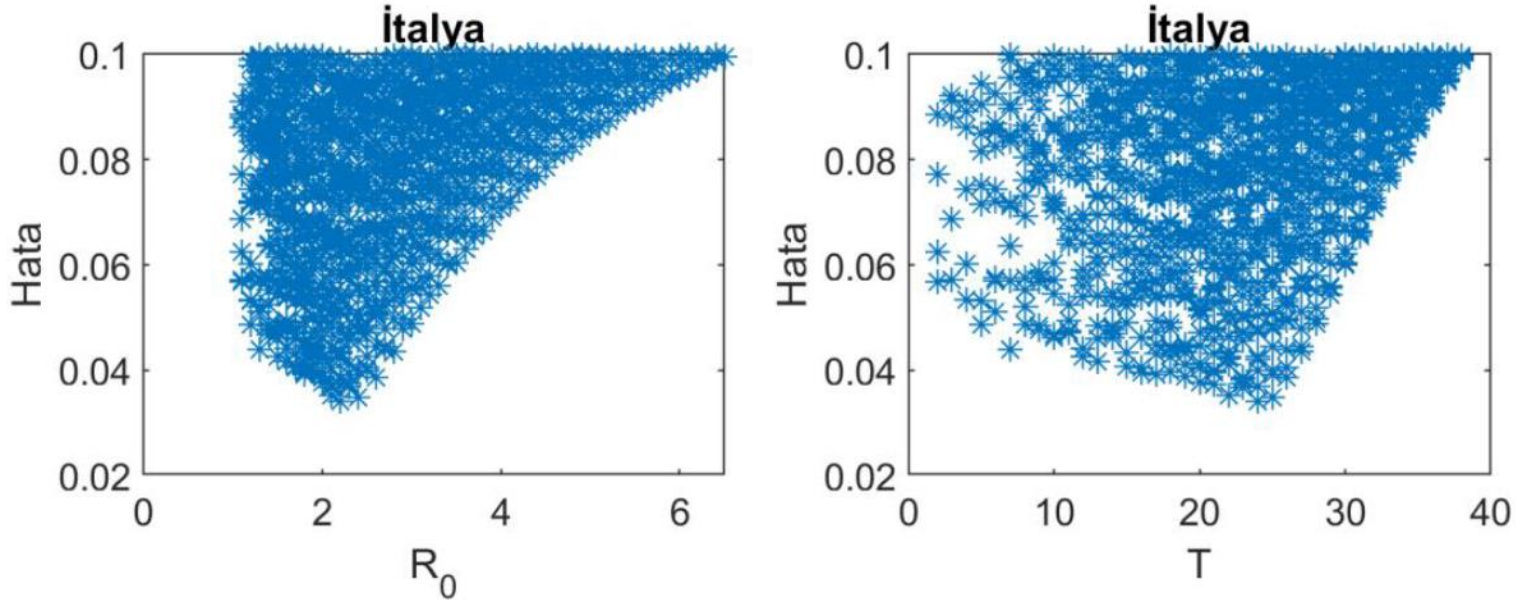




R_0 ~2.2, T~11

Data Period: 22 January 2020- 16 June 2020

COVID-19- MULTISTAGE SIR



R_0 ~2.2, T~24

Data Period: 15 February 2020- 16 June 2020

THANK YOU